IX. *On the Use of the Barometric Thermometer for the Determination of Relative Heights.* *By* James R. Christie, *Esq., of the Royal Military Academy.* *Communicated by* S. Hunter Christie, *Esq., Sec. R.S., &c.*

Received November 28, 1845,—Read January 29, 1846.

ALTHOUGH the observation of the temperature of boiling water has been for some time, but not extensively, employed for the determination of relative heights, yet the only means which experiment has confirmed of reducing it to a measure of the atmospheric pressure as usually estimated by the height of an equiponderate column of mercury has, till very recently, been overlooked; and it may perhaps be owing to this circumstance that the instrument for making the requisite observations remains to have fully developed in it the advantages it undoubtedly possesses, in portability and strength of construction, over the fragile and easily deranged barometer.

My attention having been called to this subject by a remark made by Professor Forbes in his interesting work on the Alps, to the effect that he had found the temperature of boiling water to decrease uniformly with the increase in height of the place of observation, and at the rate of one degree of Fahrenheit for every 550 feet of vertical ascent, I considered that it would be highly satisfactory to verify this result during an excursion over the Alps of Savoy and Piedmont which I then had in contemplation, and in the course of which I proposed to visit some localities at very considerable elevations above the sea level: and I was induced also to seek for some foundation for this very simple law. In prosecuting the latter inquiry, I soon found that, by assuming the truth of De Luc's formula for the determination of the boiling-point from the barometric pressure, at all accessible heights, a corroboration of the law in question is at once arrived at. I have since found, by reference to a paper in Vol. xv. of the Transactions of the Royal Society of Edinburgh, that Professor Forbes had himself verified his original conjecture in the same manner.

The formula alluded to, when reduced to the common English units of measure, becomes

$$b = \frac{99}{\cdot 899} \cdot \log 10\,\beta - 60\cdot 804 \quad \ldots\ldots\ldots\ldots \text{I.}$$

where b is the variable boiling-point on Fahrenheit's scale, and β the corresponding barometric pressure in inches of mercury. From this we obtain at once

$$\log 10\,\beta = \frac{\cdot 899}{99}\,(b + 60\cdot 804)$$

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and, calling β' and b' two other corresponding values of β and b, we have

$$\log\beta - \log\beta' = \frac{\cdot 899}{99}(b-b').$$

Substituting this value in LAPLACE's formula for the determination of H, the number of English feet in vertical height between two stations, where the barometric pressures are β and β', and the mean temperature of the air at which is t, viz.

$$H = 60345\cdot 6(\log\beta - \log\beta')\{1+(t-32^\circ)\cdot 00222\},$$

we finally get

$$H = 547\cdot 99(b-b')\{1+(t-32^\circ)\cdot 00222\} \quad \ldots\ldots\ldots \text{II.}$$

That a very high degree of accuracy attaches to the formula I. is at once evident from the observations made, in test of it, by its author, and more especially from the one made by DE SAUSSURE on the summit of Mont Blanc with the same object. I proposed therefore, in undertaking a series of observations at elevated places, to restrict myself to the question of the fitness of the barometric thermometer, in its present form, for affording trustworthy data; making observations at stations whose heights had been previously well-determined, and at as great elevations as circumstances would admit: and, in case of feeling satisfied on this point, I considered that it would be interesting to determine the extent of accuracy to which such a series of observations would give the relative levels of successive places of observation, without reference to the simultaneous observations at any fixed station.

The instrument with which my observations were made consists of a brass cylindrical boiler, 3 inches in depth and 2·25 inches in internal diameter, supported, when in use, upon a brass tripod stand; in the upper disc of this cylinder a thermometer, graduated from 181° to 215° FAHR., is made to screw by means of a strong brass collar, one inch in diameter, surrounding the tube at the distance of two inches from the bulb, which is of a pear-shaped form, having the diameter of its largest circular section ·75 inch. The whole length of the scale is rather more than twelve inches, and each degree is ·343 inch in length, subdivided into tenths, and reading, by means of a vernier, to hundredths; the index at the zero point of the vernier by enveloping the tube prevents the intrusion of parallax. The water in the boiler is heated by means of a small spirit-lamp, which I have always found sufficient for that purpose, even in situations exposed to considerable force of wind, when protected by a shade formed by coiling up and pinning together the ends of a sheet of stiff paper. The escape of steam is at a small orifice, ·15 inch in diameter, in the upper disc of the boiler, and also through the aperture left between the tube of the thermometer and its screw collar. In using the instrument I was careful to have the water as nearly as possible of the same depth on all occasions, and to make use of the purest I could procure, which was apparently always sufficiently free from any admixture of foreign matter likely to cause appreciable error in the boiling-point.

It was necessary, before deducing any results from observations made, to deter-

mine whether the instrumental graduations could be relied on; and not having had an opportunity of comparing the instrument with any standard barometer in England, I was under the necessity of employing my Geneva observations for that purpose, comparing them with the corresponding indications of the barometer at the Geneva observatory, the height of which I estimated at seventy feet above the spot at which my observations were made. This difference of level will amount to

$$\frac{70}{548} = \cdot 128^\circ \text{ Fahr.}$$

to be added to the boiling-point as deduced from the height of the barometer. These corresponding observations were

Geneva.

Observed boiling-point.	Reduced barometer.	Calculated boiling-point.
208°·820	·72129 metre.	209°·357
209·115	·72345 metre.	209·500

We have then $209\cdot357 + \cdot128 - 208\cdot820 = \cdot665$ error,
and $209\cdot500 + \cdot128 - 209\cdot115 = \cdot513$ error

$2)1\cdot178$

Mean error . . . . $= -\cdot589$

The non-accordance of the errors I am inclined to attribute to the latter pair of observations not being simultaneous, mine being made at 6 A.M. and the barometric indication being deduced from that recorded at 9 A.M., which is the earliest.

It was at once evident to me that the above error was not likely to afford a constant correction to be applied to all observations made at different points of the scale, because the lower point of graduation, being necessarily above the freezing-point, must have been determined by reference to a standard thermometer, and it was therefore not at all probable that it would differ from the truth by so large a quantity. An excellent opportunity of proving the truth of this conjecture offered itself at the Convent of the Great St. Bernard, where a register is kept of the variations of the barometric column. Arranging these as before, I obtain the results:—

Great St. Bernard.

Observed boiling-point.	Reduced barometer.	Calculated boiling-point.	Error.
197°·640	·56538 metre.	197°·709	—·069
197·680	·56630 metre.	197·787	—·107

There was a difference of level of ten or twelve feet between the two places of observation, the first having been made in the salle-à-manger of the Convent, and the other in one of the dormitories on the next floor where the barometer is fixed. The correction for this change of level is

$$-\frac{12}{548} = -\cdot022$$

to be added to the error of the first observation, which becomes then −·091; and the mean error derived from the two observations is −·099.

The instrumental error thus diminishing appears to indicate that there is a point on the scale where it becomes 0, and beyond which, as a correction, it must be applied in a negative form; and this point can readily be determined on the principle that the differences of the errors are proportional to the differences of the corresponding boiling-points; for, calling B the point on the scale at which the error vanishes, b, b', e and e' two other points and their corresponding errors, we have the equation

$$\frac{b-\mathrm{B}}{b-b'}=\frac{e\pm 0}{e-e'};$$

whence

$$\mathrm{B}=b-\frac{e.(b-b')}{e-e'};$$

and, substituting in this the values above found, viz.

$$b=208°\cdot967,\quad b'=197°\cdot649,$$
$$e=-\ \cdot589,\quad e'=-\ \cdot099,$$

we find

$$\mathrm{B}=195\cdot363.$$

In order, by means of this result, to determine the correction to be applied as a multiplier to the number of degrees above or below B, it remains to determine the amount of error due to each degree, which, in fact, when its sign is changed, will be the required factor; calling this correction C, we have

$$\mathrm{C}=\frac{-e}{b-\mathrm{B}}=\frac{\cdot589}{13\cdot604}=\cdot0433.$$

It may be remarked, that the principle on which the determination of this correction is founded remains unaffected by the supposition that the errors from which it is derived are partly attributable to the diameter of the bore of the thermometer increasing uniformly in descending; this I have satisfied myself by the usual means to be the case, though to a comparatively trifling amount.

The observations recorded at the Geneva Observatory and those at the Great St. Bernard afford an excellent means of applying my own to the determination of the height of my several stations, since those places present the advantage of being, the one my starting-point, and the other a position central to my series of stations. The readiest way of thus applying them appears to be, to reduce the height of the barometric column as given in millimètres to the form of the corresponding temperature of boiling water, by means of De Luc's formula above given, and then, for the determination of the difference of altitude between each station and Geneva, and the Great St. Bernard, to employ the formula

$$\mathrm{H}=\{548(b-b')\pm 24\mathrm{D}\}\{1+(t-32)\cdot00222\},$$

where D is the difference in degrees between B(=195·4) and the observed boiling-point; and the factor 24 is the product 548×·0433; the sign + being used when my own station is *lowest* and *vice versâ*, because the instrumental error is always one of defect.

Thus, taking the observations made on the Bonhomme and the corresponding ones at Geneva and the Great St. Bernard, we have,

Barom. Geneva. Boiling-point.
720·97 mm. gives 209·335

Obs^{d} boil^{g}-p^{t} 197°·700 197·700

195·4 11·635 × 548 = 6376

2·3 × 24 = 55 55

6321 × 1·059 = 6694 above Geneva Obs^{y}.
1334 Observ^{y} above sea.

Croix du Bonhomme above the sea 8028 feet.

Barom. G.S.B. Boiling-point.
563·94 mm. gives 197·587

197·700

·113 × 548 = 61·9

55

116·9 × 1·0357 = 121 below Great St. Bernard.
8170 Gt. St. Bernard above the sea.

Croix du Bonhomme above the sea 8049 feet.

The following Table shows the simultaneous observations at the two fixed stations and at the variable one, and the results derived from them by the above method.

Table I.

Station.	Observed boiling-point.	Temperature of the air, Fahrenheit.	Geneva barometer (reduced to 32° F.) in millimetres.	Geneva external thermometer *centigrade*.	Great St. Bernard barometer (reduced to 32° F.) in millimetres.	Great St. Bernard external thermometer *centigrade*.	Calculated height in feet above Geneva observatory.	Calculated depression in feet below the Great St. Bernard.	Sum of the two preceding (6836 true).	Height in feet above the sea by Geneva observation.	Height in feet above the sea by G.S.B. observatory.	Height in feet as previously determined.	Authority for the preceding column.
Les Rousses	204°·48	57°	722·01	19°·4	566·66	5°·9	2644	4024	6668	3978	4146	3919	Woerl.
Geneva	208·82	71	721·29	19·5	563·98	8·2	(−70)	6862	6792	.....	1308	1334	Bib. Univ. de Génève.
Bonneville	208·715	66	722·69	21·7	564·14	6·4	90	6730	6820	1424	1440	1452	Saussure.
Môle (summit)	199·855	53	723·03	19·7	565·00	4·0	5519	1300	6819	6853	6870	6221	Pictet.
St. Jeoire	208·40	62	724·54	17·9	565·73	5·3	352	6424	6776	1686	1746	1778	Keller.
Samoens	207·94	55·5	727·61	17·0	566·91	2·5	747	6017	6764	2085	2153	2257	Woerl.
Col Dichaud	198·24	52	727·73	23·9	568·46	7·3	6695	224	6919	8029	7946		
Servoz	207·31	58	729·06	15·5	569·25	4·0	1171	5571	6742	2505	2599		
Chamounix—1.	205·70	66·5	728·50	20·3	569·44	10·0	2174	4695	6869	3508	3475	3364	Saussure.
Chamounix—2.	205·50	50·5	723·91	17·6	568·38	5·1	2076	4663	6739	3410	3507	3425	Forbes.
Chamounix—3.	205·52	68	721·40	15·4	566·11	9·2	1996	4751	6747	3330	3419	3408	Woerl.
Montanvert	200·27	62	722·57	22·1	568·19	9·6	5286	1464	6750	6620	6706	6300	Forbes.
Contamines—1.	204·71	63	722·61	19·3	565·80	5·5	2544	4223	6767	3877	3947	4260	Woerl.
Contamines—2.	204·72	62	722·41	15·4	564·33	6·5	2561	4305	6866	3896	3865	3259	Keller.
Croix du Bonhomme	197·70	49	720·97	19·9	563·94	8·5	6694	131	6825	8028	8049	8026	Saussure.
Chapiu—1.	202·35	54	721·67	15·5	563·55	3·4	3889	2883	6772	5223	5287	5071	Saussure.
Chapiu—2.	202·655	52	724·52	17·5	563·02	1·2	3821	3069	6880	5155	5101	5006	Woerl.
Bourg St. Maurice	207·00	59	726·97	16·5	565·59	1·2	1295	5534	6829	2629	2636		
Hospice Pt. St. Bernard	199·24	50	724·47	22·9	566·64	9·0	5870	905	6775	7203	7265	7194	Saussure.
Cormayeur—1.	204·75	57	723·74	16·0	565·69	4·3	2441	4336	6777	3775	3834	3997	Saussure.
Cormayeur—2.	204·60	55·5	721·98	12·8	562·32	4·2	2535	4280	6815	3869	3890	4167	Forbes.
Cormayeur—3.	204·36	54	724·81	17·3	565·61	7·3	2806	3990	6796	4140	4180	4046	Woerl.
Aosta—1.	208·59	73	724·19	19·4	565·72	8·7	225	6651	6876	1561	1519	1938	Saussure.
Aosta—2.	208·46	61	721·74	14·2	561·61	+ ·2	205	6581	6786	1539	1589		
Aosta—3.	208·35	68	722·23	15·9	561·26	+ 1·8	293	6611	6904	1627	1559		
Aosta—4.	208·59	57	726·08	15·3	563·75	− 2·1	293	6505	6798	1627	1665		
Aosta—5.	208·56	72	726·41	20·4	567·18	+ 6·6	361	6542	6903	1695	1628		
St. Remy	202·545	52	725·55	17·4	564·82	+ 0·6	3928	2913	6841	5262	5257	5263	Saussure.
Great St. Bernard—1.	197·64	39	725·23	17·4	565·27	+ 2·7	6791	.....	6791	8125	.....	8170	Bib. Univ. de Génève.
Great St. Bernard—2.	197·68	38	726·42	17·1	566·67	6·5	6753	.....	6753	8087	.....	8038	Saussure.
St. Vincent—1.	209·28	63	727·96	15·9	568·84	3·6	− 54	6785	6731	1280	1385		
St. Vincent—2.	209·64	55	730·25	20·0	569·04	2·8	− 184	6799	6615	1150	1371		
Col de Jou	202·82	56	730·25	21·0	569·99	5·8	3984	2871	6855	5318	5299		
Brussone	204·65	66·5	730·25	22·8	570·41	7·3	2904	3740	6644	4238	4430	4431	Forbes.
Col Ranzola	199·45	53	728·99	24·3	570·59	7·9	6012	843	6855	7346	7327	7136	Forbes.
Gressonay—1.	204·06	58·5	730·52	18·0	570·94	4·3	3224	3681	6905	4558	4489	4208	Saussure.
Gressonay—2.	204·33	58·5	730·15	18·4	570·29	6·1	3050	3766	6816	4384	4404	4526	Woerl.
Gressonay—3.	203·92	57	730·37	16·9	568·27	2·6	3294	3752	7046	4628	4418		
Col Val Dobbia	197·44	52	730·40	19·4	568·75	2·8	7226	− 262	6964	8560	8432	7904	Saussure.
La Riva—1.	205·54	61	729·94	15·8	569·74	2·5	2299	4494	6793	3633	3676	3568	Saussure.
La Riva—2.	205·55	55	727·70	18·0	569·40	7·0	2200	4527	6727	3534	3643	3636	Woerl.
Varallo—1.	209·36	67	728·18	20·0	570·86	7·4	− 96	6814	6718	1238	1356		
Varallo—2.	209·26	63	727·82	20·0	570·52	6·2	− 47	6727	6680	1287	1443	1491	Woerl.
La Colma	206·72	70	728·20	27·1	571·67	13·8	1560	5263	6823	2894	2907		
Orta—1.	210·70	74	729·11	21·5	572·34	8·5	− 895	7621	6726	439	549	1108	Woerl.
Orta—2.	210·83	70	729·79	22·9	571·80	11·2	− 921	7628	6707	387	518		
Orta—3.	210·35	66	728·80	18·0	569·67	5·6	− 681	7452	6771	653	718		
Mogadino—1.	211·04	77	727·80	19·4	568·60	5·2	−1146	7985	6839	188	185	678	Saussure.
Mogadino—2.	211·20	68	724·55	18·0	566·74	6·0	−1368	8110	6742	− 34	+ 60		
Domo d'Ossola—1.	210·68	78	723·38	17·4	565·64	5·9	−1104	7933	6829	230	237	1004	Saussure.
Domo d'Ossola—2.	210·44	66	723·27	16·0	563·89	4·5	− 943	7751	6808	391	419		
Simplon Village	203·28	57	724·03	17·0	564·12	7·4	3440	3441	6881	4729	4774	4792	Woerl.
Simplon Hospice	201·07	48	724·00	19·0	564·13	5·3	4753	2084	6837	6087	6086	6580	Saussure.
Brieg—1.	208·69	61	726·01	11·4	564·45	+ 1·6	229	6606	6835	1563	1564	2328	Saussure.
Brieg—2.	208·35	56	729·28	15·0	566·00	− 1·3	552	6194	6746	1886	1976	2183	Woerl.
Leukerbad—1.	204·38	47·5	733·42	14·6	570·43	− 1·2	3104	3673	6777	4438	4497	4699	Woerl.
Leukerbad—2.	204·545	45	734·75	13·7	571·54	+ 2·0	3025	3749	6774	4359	4421		
S. Crest of the Gemmi	199·235	41	734·44	19·7	572·06	4·7	6235	620	6855	7569	7550	7626	Woerl.
Kandersteg—1.	206·34	55·5	733·06	16·3	572·45	3·0	1922	4816	6738	3256	3354	3493	Woerl.
Kandersteg—2.	206·17	48	732·09	15·8	571·74	9·3	1972	4765	6737	3306	3405		
Lauterbrunnen—1.	208·28	55	728·81	16·6	570·66	8·8	585	6112	6697	1918	1958		
Lauterbrunnen—2.	208·37	49	727·68	17·6	569·18	10·3	484	6215	6699	1818	1955		
Bern	209·26	72	726·97	22·4	569·91	11·0	− 82	6883	6801	1252	1287	1661	Woerl.

The discrepancies in many of the above results as compared with the column of previously determined heights are considerable, but it is to be observed that in most cases where the height of the station may be considered as particularly well-established, the observed height accords with it in a very remarkable manner. This is particularly the case at Geneva (where the height is of course deduced only from that of the Great St. Bernard), at Bonneville, at Chamounix (taking a mean of the six observations, which gives 3442), at the Croix du Bonhomme*, at St. Remy, at the Great St. Bernard and at Brussone: the chief exceptions being at the Môle†, the Montanvert, the Col di Val Dobbia, Orta, Mogadino, Domo d'Ossola, the Simplon Hospice and Brieg, the great differences at which places are to me quite unaccountable, unless on the supposition, in the case of the five last-named, that their distances from the fixed stations were too great to allow of trustworthy results being obtained from single observations. In confirmation of this view, it will be seen presently that these observations give far more satisfactory results when employed without any reference to the simultaneous observations at Geneva and the Great St. Bernard; and also in the above Table we may notice that, after crossing the Bonhomme, in those cases where the heights determined from the two fixed stations differ from one another to any considerable amount, that obtained by reference to the Great St. Bernard generally approximates more nearly to the height as previously determined than the other, obtained by a comparison with the more distant station.

The numbers in the tenth column which are the sums of the two values, the height above Geneva and the depression below the Great St. Bernard, as determined by observation, should, supposing both theory and observations to be perfect, give the constant value 6836, which is the difference of height of those two stations; but this difference will vary when determined by means of each pair of barometric observations, the standard value 6836 being the mean of a very considerable number. Now it is very evident that were it not for that term in the formula which involves the correction for the expansion of the air, the sum of the above values would give the difference of altitude, however erroneous the intermediate observation might be: for calling b', b'' and b''' the three boiling-points, we should have

$$548(b'-b'')-24\mathrm{D}+548(b''-b''')+24\mathrm{D}=548(b'-b''').$$

The observation, therefore, of the external temperature at the intermediate station becomes the means of rendering the accordance of the sums of the component altitudes with the whole height between Geneva and the Great St. Bernard, as determined in each case, a test of the correctness of the observation of the boiling temperature. We may in fact, by equating the formula for the whole height with the sum

* All who have traversed the Pass of the Bonhomme are aware that in crossing to the south, upon Chapiu, the tract rises considerably after passing what ought to be and, I believe, *is* considered the "*Col*," viz. the point from which a view of the mountains of the Tarentaise is first obtained; this point is marked by a cross, larger than usual on most passes, and than any of the numerous others on this pass. This cross I believe to be the "*Croix du Bonhomme*" of De Saussure, and it was at its base the observation above referred to was made.

† See note, p. 131.

of those expressing the two component heights, arrive at an expression for the intermediate boiling-point in terms of the atmospheric temperatures at the three stations and the upper and lower boiling-points; such an expression is

$$b'' = \frac{23}{24} \cdot \left\{ \frac{t-T'}{T-T'} \cdot b' + \frac{T-t}{T-T'} b''' + 8{\cdot}4 \right\}$$

where T, t and T' are the temperatures at the stations where the boiling-points are respectively b', b'' and b'''; but this, when put in the form

$$b'' = \frac{23}{24} \cdot \left\{ \frac{t.(b'-b''') + Tb''' - T'b'}{T-T'} + 8{\cdot}4 \right\},$$

shows us that an error in the value of t, of trifling moment where that value is to be employed in the determination of the difference of level, will cause an error in the resulting boiling temperature of very serious amount when used for the same purpose. Though this test cannot therefore be made use of in its full rigour, it will be highly satisfactory if an accordance can be shown between the results above referred to: the following, Table II., exhibits a comparison of these values. The numbers in the second column are the differences of the boiling temperatures, in degrees of FAHRENHEIT, deduced from the barometric observations at Geneva and the Great St. Bernard, which were simultaneous with those of my own made at the places named in the first column.

TABLE II.

Station.	Difference of boiling-points derived from the observations at Geneva and the Great St. Bernard.	Value of $(t-32°)$ in formula II.	Derived difference of height.	Difference of height from Table I.	Station.	Difference of boiling-points derived from the observations at Geneva and the Great St. Bernard.	Value of $(t-32°)$ in formula II.	Derived difference of height.	Difference of height from Table I.
Les Rousses	11°·583	22°·8	6669	6668	Col de Jou	11°·860	24°·1	6847	6855
Geneva				6792	Brussone	11·372	27·0	6605	6644
Bonneville	11·856	25·3	6862	6820	Col Ranzola	11·716	29·0	6833	6855
Môle	11·794	21·4	6768	6810	Gressonay—1.	11·789	20·0	6719	6905
St. Jeoire	11·832	20·9	6784	6776	Gressonay—2.	11·818	22·0	6763	6816
Samoens	11·936	17·6	6796	6764	Gressonay—3.	12·003	17·5	6833	7046
Col Dichaud	11·813	28·1	6877	6919	Col di Val Dobbia	11·964	20·0	6847	6964
Servoz	11·834	12·6	6666	6742	La Riva—1.	11·851	16·5	6735	6793
Chamounix—1.	11·781	27·3	6846	6869	La Riva—2.	11·729	22·5	6749	6727
Chamounix—2.	11·568	20·5	6627	6739	Varallo—1.	11·640	24·7	6728	6718
Chamounix—3.	11·593	22·2	6664	6747	Varallo—2.	11·646	23·6	6716	6680
Montanvert	11·491	28·6	6696	6750	La Colma	11·574	26·8	6860	6823
Contamines—1.	11·699	22·3	6728	6767	Orta—1.	11·578	27·0	6725	6726
Contamines—2.	11·811	19·7	6755	6866	Orta—2.	11·661	25·7	6758	6707
Croix du Bonhomme	11·748	25·6	6803	6825	Orta—3.	11·782	21·3	6761	6771
Chapiu—1.	11·828	17·0	6726	6772	Mogadino—1.	11·805	22·1	6786	6839
Chapiu—2.	12·061	16·9	6857	6880	Mogadino—2.	11·748	21·6	6746	6742
Bourg St. Maurice	12·005	16·0	6811	6829	Domo d'Ossola—1.	11·764	20·9	6746	6829
Hospice Pt. St. Bernard	11·642	28·7	6786	6775	Domo d'Ossola—2.	11·865	18·5	6762	6808
Cormayeur—1.	11·748	25·6	6803	6777	Simplon Village	11·936	22·0	6859	6881
Cormayeur—2.	11·953	15·4	6775	6815	Simplon Hospice	11·933	20·9	6842	6837
Cormayeur—3.	11·861	27·1	6852	6796	Brieg—1.	12·038	11·7	6768	6835
Aosta—1.	11·809	25·3	6835	6876	Brieg—2.	12·121	12·4	6826	6746
Aosta—2.	11·997	13·0	6785	6786	Leukerbad—1.	12·020	12·1	6763	6777
Aosta—3.	12·060	15·9	6842	6904	Leukerbad—2.	12·014	14·2	6790	6774
Aosta—4.	12·102	13·2	6806	6798	South Crest of the Gemmi	11·950	22·0	6868	6855
Aosta—5.	11·895	24·3	6835	6903	Kandersteg—1.	11·828	17·4	6731	6738
St. Remy	11·976	16·2	6798	6841	Kandersteg—2.	11·823	22·6	6804	6737
Great St. Bernard—1.	11·911	18·1	6790	6791	Lauterbrunnen—1.	11·700	22·9	6737	6697
Great St. Bernard—2.	11·873	21·3	6816	6753	Lauterbrunnen—2.	11·749	25·1	6797	6699
St. Vincent—1.	11·795	16·5	6715	6731	Bern	11·641	30·1	6805	6801
St. Vincent—2.	11·930	20·5	6835	6615					

It is to be observed, that the differences of height determined by the two methods do not differ from one another in any single case by so large a quantity as do the greatest and least difference of level in the fourth column, determined from the barometric observations at the two places; while in many the accordance is almost perfect. One of the most satisfactory results in this respect is, however, that derived from the observation No. 2. at Mogadino, which appears from Table I. to be one of those least likely to be correct, since it places the surface of the Lago Maggiore much below its possible level. This alone sufficiently shows that the accordance above remarked can only be taken as a general indication of the accuracy of the instrument.

Having thus shown that the instrument is perfectly capable of furnishing correct data for the determination of difference of level, though apparently its indications cannot in all cases be relied on, it becomes further interesting to inquire how far it may be depended on when made use of without reference to corresponding observations at any fixed station. It is obvious that the accuracy of a series of heights thus obtained must, to a certain extent, be vitiated by the diurnal changes in the atmospheric pressure, but it will be seen that this source of error does not operate to the extent that might, *à priori*, be supposed probable, and that, whether from a fortuitous balancing of errors, or from some other cause, the discrepancies in those cases where the previously determined height is probably most near the truth, are less considerable than might be expected. In order thus to obtain the results given in the following Table III., it became necessary again to modify the standard formula on account of the instrumental error. By multiplying the correction, C=·0433, by 548 we obtain it in the form of a difference of altitude for each degree of observed difference of boiling temperature at two successive places of observation, to be added to the factor 548 to form the constant multiplier for this particular instrument. We thus obtain

$$548 \times 1{\cdot}0433 = 571{\cdot}72,$$

or, with sufficient accuracy, 572.

The working formula then becomes

$$H = 572(b - b').\{1 + (t - 32^\circ){\cdot}00222\}.$$

In the following Table, in which the results have been calculated with this formula, the differences of level are determined from station to station in the order in which the observations were made, and, in case of two or more observations having been made at the same station, those most approximate in time have been employed. Taking the station at Geneva as a starting-point, and assuming its level, derived from that of the surface of the lake given in the Bibliothèque Universelle de Génève, to be 1264 English feet above the sea level, the calculated differences have been added to this in succession, and the altitudes by observation obtained.

Table III.

Station.	Time of observation, 1844.	External temperature of the air.	Observed boiling-point.	Dif. of altitude in ft.	Altitude in feet.	Altitude as previously determined.	Authority for the altitudes in the preceding column.
	h m	°					
Les Rousses (Jura)	June 25. 5 30 A.M.	57	204·48	2767	4031	3919	Woerl.
Geneva—1.	25. 0 30 P.M.	71	208·82	2767	....	1264	Bib. Univ. de Génève.
Geneva—2.	26. 6 15 A.M.	62	209·115				
Bonneville	26. 11 45 A.M.	66	208·715	247	1511	1452	Saussure.
Môle	26. 6 15 P.M.	53	199·855	5388	6899	6094 Saussure. / 6221 Pictet.	
St. Jeoire	27. 8 0 A.M.	62	208·40	5166	1733	1893	De Luc.
Samoens	28. 5 30 A.M.	55·5	207·94	278	2011	2270	De Luc.
Col Dichaud (Buet)	28. 3 45 P.M.	52	198·24	5816	7827		
Servoz	29. 7 30 A.M.	58	207·31	5453	2374		
Chamounix—1.	29. 1 45 P.M.	66·5	205·70	983	3357	3364	Saussure.
Chamounix—2.	30. 4 30 A.M.	50·5	205·50	....	....	3425	Forbes.
Montanvert	30. 0 15 P.M.	62	200·27	3152	6509	6101 Saussure. / 6300 Forbes.	
Chamounix—3.	30. 4 20 P.M.	68	205·52	3223	3286	3356	Pictet.
Contamines	July 1. 7 15 P.M.	63	204·71	500	3786	3328	Keller.
	2. 9 15 A.M.	62	204·72	....	....	4260	Woerl.
Croix du Bonhomme	2. 3 30 P.M.	49	197·70	4269	8055	8026	Saussure.
Chapiu—1.	2. 7 30 P.M.	54	202·35	2775	5280	5071	Saussure.
Chapiu—2.	3. 9 20 A.M.	52	202·655	....	....	4974	Pictet.
Bourg St. Maurice	3. 8 0 P.M.	59	207·00	2566	2714		
Petit St. Bernard	4. 11 45 A.M.	50	199·24	4660	7374	7194	Saussure.
Cormayeur—1.	4. 9 30 P.M.	57	204·75	3302	4072	3997	Saussure.
Cormayeur—2.	7. 7 0 A.M.	54	204·36	....	....	4167	Forbes.
Aosta—1.	7. 1 45 P.M.	73	208·59	2589	1483	1938	Saussure.
Aosta—2.	9. 5 30 A.M.	57	208·59	....	....	1980	Woerl.
St. Remy	9. 11 45 A.M.	52	202·545	3631	5114	5263	Saussure.
Grand St. Bernard—1.	9. 3 30 P.M.	39	197·640	2891	8003	8038	Saussure.
Grand St. Bernard—2.	10. 6 30 A.M.	38	197·680	....	....	8170	Bib. Univ. de Génève.
Aosta—3.	10. 0 30 P.M.	72	208·560	6541	1462		
St. Vincent—1.	10. 9 0 P.M.	63	209·280	444	1018		
St. Vincent—2.	11. 5 30 A.M.	55	209·64				
Col de Jou	11. 10 30 A.M.	56	202·82	4148	5166		
Brussone	11. 0 0 noon.	66·5	204·65	1109	4057	4431	Forbes.
Col de Ranzola	11. 5 30 P.M.	53	199·45	3157	7214	7136	Forbes.
Gressonay—1.	11. 7 45 P.M.	58·5	204·06	2776	4438	4208	Saussure.
Gressonay—2.	13. 8 15 A.M.	57	203·92	....	....	4526	Woerl.
Col di Val Dobbia	13. 1 0 P.M.	52	197·44	3892	8330	7904	Saussure.
La Riva—1.	13. 7 30 P.M.	61	205·54	4885	3445	3568	Saussure.
La Riva—2.	14. 6 0 A.M.	55	205·55	....	....	3712	Woerl.
Varallo—1.	14. 10 0 P.M.	67	209·36	2319	1126	1491	Woerl.
Varallo—2.	15. 6 45 A.M.	63	209·26				
La Colma	15. 2 15 P.M.	70	206·72	1566	2692		
Orta—1.	15. 6 45 P.M.	74	209·70	1856	836	1108	Woerl.
Orta—2.	17. 4 30 A.M.	66	210·35				
Mogadino—1.	17. 6 30 P.M.	77	211·04	430	406	678	Saussure.
Mogadino—2.	18. 6 0 A.M.	68	211·20				
Domo d'Ossola—1.	18. 7 0 P.M.	78	210·68	324	735	1004	Saussure.
Domo d'Ossola—2.	19. 4 30 A.M.	66	210·44				
Simplon (Village)	19. 0 30 P.M.	57	203·28	4368	5098	4792	Woerl.
Simplon Hospice	19. 4 50 P.M.	48	201·07	1323	6421	6580	Saussure.
Brieg—1.	19. 10 30 P.M.	61	208·69	4581	1840	2328	Saussure.
Brieg—2.	20. 7 15 A.M.	56	208·35	....	....	2183	Woerl.
Leukerbad—1.	20. 8 0 P.M.	47·5	204·38	2371	4211	4697	Woerl.
Leukerbad—2.	21. 7 0 A.M.	45	204·545				
Gemmi (châlet on south crest)	21. 0 30 P.M.	41	199·235	3112	7323	7626	Woerl.
Kandersteg—1.	21. 6 30 P.M.	55·5	206·34	4208	3115	3493	Woerl.
Kandersteg—2.	22. 6 45 A.M.	48	206·17				
Lauterbrunnen—1.	22. 9 0 P.M.	55	208·28	1251	1864		
Lauterbrunnen—2.	23. 6 45 A.M.	49	208·37				
Bern	24. 10 20 A.M.	72	209·26	536	1328	1661	Woerl.

Though the discrepancies in these results, when compared with the previously determined altitudes, appear very considerable, they are probably in most cases not more so than would have resulted from the employment of the usual form of barometer. I am led to this conclusion from the inspection of De Luc's series of barometric measurements, wherein the determinations of the height of the same station at different times frequently differ from one another 250 feet, and, in one case, as much as 360 feet. The greatest error in the above table, viz. that at Aosta*, 400 feet, certainly exceeds either of these, but there is, I think, good reason to suppose that the observed height at Aosta is not so erroneous as the comparison with De Saussure's result makes it appear to be, when we remark that the height of the Great St. Bernard determined from it is one of the most accurate of the series, and also that the observations made *the next day* at the Great St. Bernard and at Aosta give the same difference of level, within twenty feet.

The only other instance in which I was enabled to repeat an observation after the ascent of any considerable height occurred at Chamounix, and in it a tolerably close accordance in the observations before and after the ascent of the Montanvert is also to be remarked, though the height derived from the intermediate observation differs very considerably even from that given by Professor Forbes, which is however equally in excess above that by De Saussure.

Since the calculation of the above results I have had an opportunity of comparing the instrument with a standard barometer, and have been surprised to find the instrumental correction, under the pressure 30 inches, very different from that obtained above from the Geneva observations. I cannot assign any reason for supposing an error in making these observations; indeed, every consideration induces me to believe that no such error could have occurred, but at the same time I find it very difficult to account for this change in the correction, unless it be partly attributable to a difference in construction of the two barometers and partly to a difference in the degree of purity of the water, or to a change in the instrument itself; all of which suppositions are highly improbable. To whichever of the above causes this remarkable change is referred, I do not think it should be considered as vitiating the results derived from the recorded observations, since they have been connected with those made with the barometers by means of which the correction made use of has been determined; and moreover the time elapsed between the first and last recorded observations was only one month, and the sources from which the water was derived were in all cases so far similar as apparently to ensure its chemical identity with respect to boiling temperature.

The utility of an instrument to be applied to purposes which involve the necessity

* The circumstances of danger under which the observation on the Môle was made, make me think that an erroneous reading is not unlikely to have occurred there, and I accordingly exclude it from consideration in these remarks.

of its removal from place to place under difficult circumstances, is not *exclusively* to be measured by the degree of delicacy or even of accuracy of its indications; since however perfect it may be theoretically, when stationary, it can never be of practical benefit unless it be of such a construction as to bear the concussions and shaking it must be necessarily exposed to when conveyed in the manner in which it can alone arrive at the point where its agency is required. This construction has been attained in the instruments by means of which are determined the two principal coordinates of geographical position, under all circumstances involving difficulty of access to the place of observation, while the third, *height above the sea level*, has had almost exclusively applied to its determination an instrument more liable than almost any other to suffer during removal, and, from its very nature, incapable of any great improvement in this respect. With regard to the instrument under consideration, I can, in reference to the above points, speak most positively; not only were the concussions which it constantly suffered very considerable, but on two or three occasions it escaped uninjured by blows which must have shivered any common barometer, however externally protected; and it has only at length been destroyed through having been inadvertently left boiling;—the water having evaporated, the increase of temperature of the empty boiler proved too great for the capacity of the tube of the thermometer, and the bulb burst.

Since writing the above my attention has been directed to the researches of Holtzmann and Magnus, given in Vol. iv. Part 14. of the "Scientific Memoirs," and I have applied the formula for the expansive force of steam deduced by the former, to the determination of an expression for the difference of level in terms of the difference of boiling temperature. Reduced to English measures, Holtzmann's result gives

$$b-32^\circ=425\cdot2\times\frac{\log\beta+\cdot74883}{6\cdot73157-\log\beta}, \quad \text{III.}$$

b and β being the same as in (I.).

Applying this as before, in the case of De Luc's, I obtain

$$H=\frac{191939000}{(393\cdot2+b)(393\cdot2+b')}\cdot(b-b')\{1+(t-32^\circ)\cdot00222\} \quad \text{IV.}$$

The results derived from (III.), within the limits of my observations, differ but little from those obtained from (I.), but have a tendency to diminish the boiling temperature more rapidly as the pressure decreases, as the following results sufficiently show:—

Pressure.	Temperature.		
	De Luc.	Holtzmann.	Diff.
721·74 mm.	209·386	209·425	+·039
561·61 mm.	197·389	197·237	−·152

Even were this last difference greater, we must still rely upon De Luc's until it can be shown that the results obtained with (III.) are more nearly in accordance with observations made under natural atmospheric pressures than those derived from (I.) have been found to be.

The practical disadvantages of (IV.) are evident.